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DESIGN OPTIMIZATION FOR A VIBRATING VERTICAL CANTILEVER

S. D. Gooch, J. K. Raine and H. McCallion

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1. Introduction

There are numerous practical uses for flexible cantilever beams. These include springs, fishing rods and radio antenna. Flexible cantilevers are also found in nature e.g. tree trunks. This paper investigates the particular case of a flexible vertical cantilever used to make a kinetic sculpture.

While the vibration of vertical cantilevers has been thoroughly investigated, exact solutions including self weight load (significant in the design of a flexible vertical cantilever) have only recently been obtained, Naguleswaran [1]. This paper investigates influences of size on the structural properties and the design implications for the kinetic sculpture, Blade.

Len Lye (1901 - 1980) built the kinetic sculpture, Blade (1965), as a prototype for what he perceived to be a much larger work. Lye's original Blade consisted of a blade of cold rolled carbon steel strip measuring 1630 x 200 x 1.85mm. The blade, a vertical cantilever, is fixed in a rigid clamp that has a linear reciprocating base motion. In front of the blade stands a wand-mounted cork ball. The imposed base motion causes a large amplitude vibration in the blade; the blade interacts with the wand and results in the artist's desired aesthetic and acoustic performance.

The objective of this paper is to determine the maximum practical economic size to which a vibrating vertical cantilever, in the form of the kinetic sculpture Blade, can be built using currently available materials and technology.

2. Design requirements

Len Lye's design requirements in the performance of Blade primarily concern the aesthetic and acoustic characteristics of the work, as experienced by the audience. He required the drive mechanism and the support structure to be concealed within a cylindrical base cover. The new owner of the sculpture, while primarily concerned with the preserving the artists wishes, also imposed cost constraints. From a design engineering perspective, attributes influencing the structural feasibility were important. In order to negotiate the specifications for the new work, a Blade design requirement specification was developed. This requirements specification was formulated as a list of demands & wishes in accordance with the design procedure of Pahl & Beitz (1996). A subset of the Blade design requirement specification is recorded in Table 1.

Gooch [3] showed that the mechanism and support structure could be built to provide sufficient power and support for the blade while operating within the space constraints specified in Table 1. While the operating environment (out doors) and noise emission requirements for the mechanism require special consideration, they are not considered to be factors limiting the size of the sculpture. The design constraints concern the blade only (i.e. the vertical cantilever), these may be summarised and considered under into the four sub headings given in Figure 1.

Table 1. Blade design requirement specification

Demand Wish	*Blade performance requirement specification 'the artists aesthetic and acoustic requirements'*
D	geometric and static similarity to be preserved in changing blade size
D	sound quality for the blade material better than would be expected from a mild steel blade *($\eta < 2 \times 10^{-3}$ @ 30°C)*
D	running costs for a nominally *2 – 2.5* times prototype size blade to be less than *500 NZ$/performance*
D	mechanism to be concealed in black cylinder *(maximum outside diameter = $0.6 \times l_o$ and maximum height $0.375 \times l_o$)*
W	infinite design life for the blade
W	low noise emission from the drive mechanism *(< 50 dB-A)*

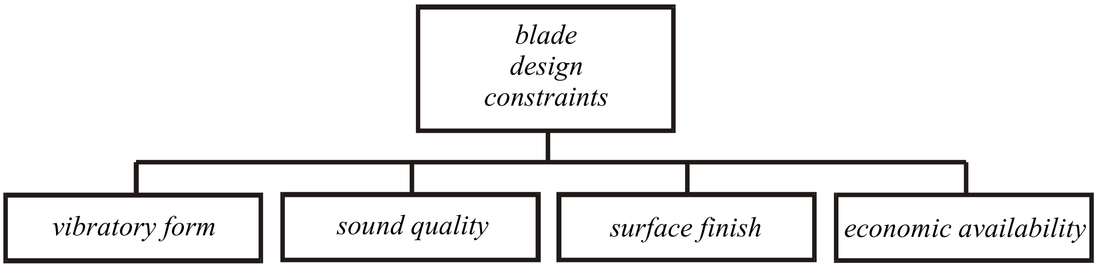

Figure 1. Design constraints for Blade

Considering the characteristic groups identified in Figure 1 the approach adopted for obtaining an optimal design in this paper is the 'method of optimum design' (after Johnson [4]). The optimum solution will yield the maximum size vertical cantilever within the limits imposed by the design requirement specification, Table 1.

3. Method of optimal design for the vibrating vertical cantilever

Using the 'method of optimum design', satisfaction of the design constraints of Figure 1 may be pursued. The constraints will be considered under two headings, equality constraints, being conditions that must be met for the project to be successful and, inequality constraints, being the factors that have upper or lower limits.

3.1 The vibratory form

Equality constraints require that geometric similarity of the vibratory blade form exist between Len Lye's original sculpture *(the model)* and the larger size work *(the prototype)*. This implies

$$V\left(\frac{x}{l}\right)_p = \frac{l_p}{l_m} V\left(\frac{x}{l}\right)_m \tag{1}$$

where: V is the lateral beam displacement
l is the beam length; x is the distance along the beam
subscripts m and p correspond to the model and prototype respectively.

While the artists brief requires geometric similarity for the length and width dimension, the blade thickness is free within limits of aesthetic acceptance. To preserve the artistic integrity of the work requires static similarity, that is geometric similarity of a loaded system. For the particular case of a vertical cantilever, Gooch [3] derived the thickness *(d)* constraint for static similarity as

$$d = \sqrt{\frac{2.12\rho g l^3}{E}} \tag{2}$$

From (2) it can be seen that increasing the size of the blade, using the same material properties, requires the blade thickness to increase as $l^{3/2}$. For the same geometrically displaced blade shape (i.e. the same local curvature) this results in an increase in bending stress. Gooch [3] found that the worst-case scenario for the assumed vibratory form, is likely to be a superposition of the first and third blade bending modes *(with a maximum blade displacement of 0.435 & 0.150m for the first and third modes respectively)*. This worst-case scenario is shown in Figure 2.

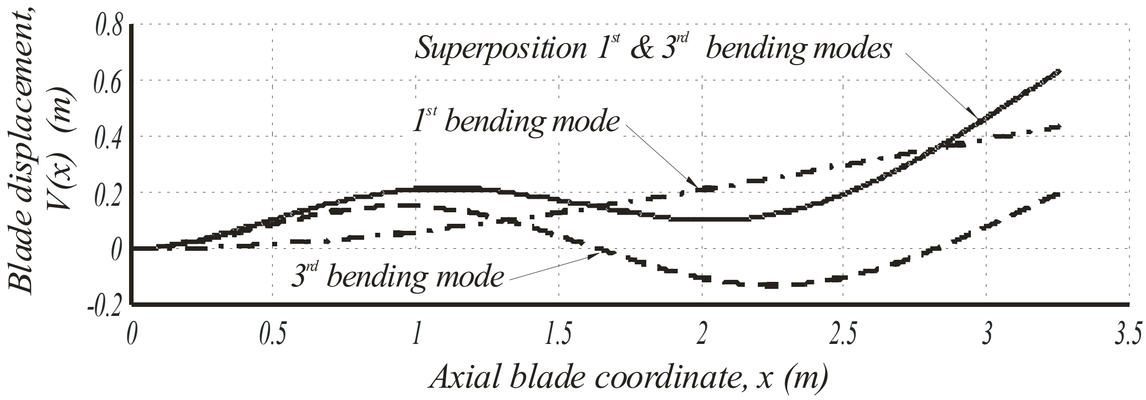

Figure 2. The assumed worst-case vibratory blade form

The minimum radius of curvature and corresponding maximum bending stress for the vibratory form in Figure 2 is at the fixed end *(x = 0)* of the cantilever. For the case of the kinetic sculpture blade, it was found that a reversed bending stress will be imposed at a level expected to result in a finite working life for the blade.

3.2 Sound Quality

Like a good bell or percussion instrument, for minimum energy loss a vibrating vertical cantilever must have good reverberant qualities and requires a material with low intrinsic damping or internal friction. Intrinsic damping is an important property when structures vibrate and is characterised by the loss coefficient η. Figure 3 shows the loss coefficient η plotted against elastic modulus for a range of engineering materials. The high carbon steel blade used on the original sculpture has a particularly low loss coefficient, $(\eta = 10^{-4})$, as do copper alloys and glass which have been traditionally used as bell materials. Titanium alloy, alloy steel, aluminium alloy, have a higher loss coefficient than high carbon steel, thus a deterioration in sound quality would be expected for the sculpture using a blade made from these materials.

Following an experimental investigation (Gooch [3]) it was decided to set an upper limit on the damping coefficient of candidate blade materials. The upper limit defined is the sound quality that would be expected from a mild steel blade material requiring

$$\eta \leq 2 \times 10^{-3} \tag{3}$$

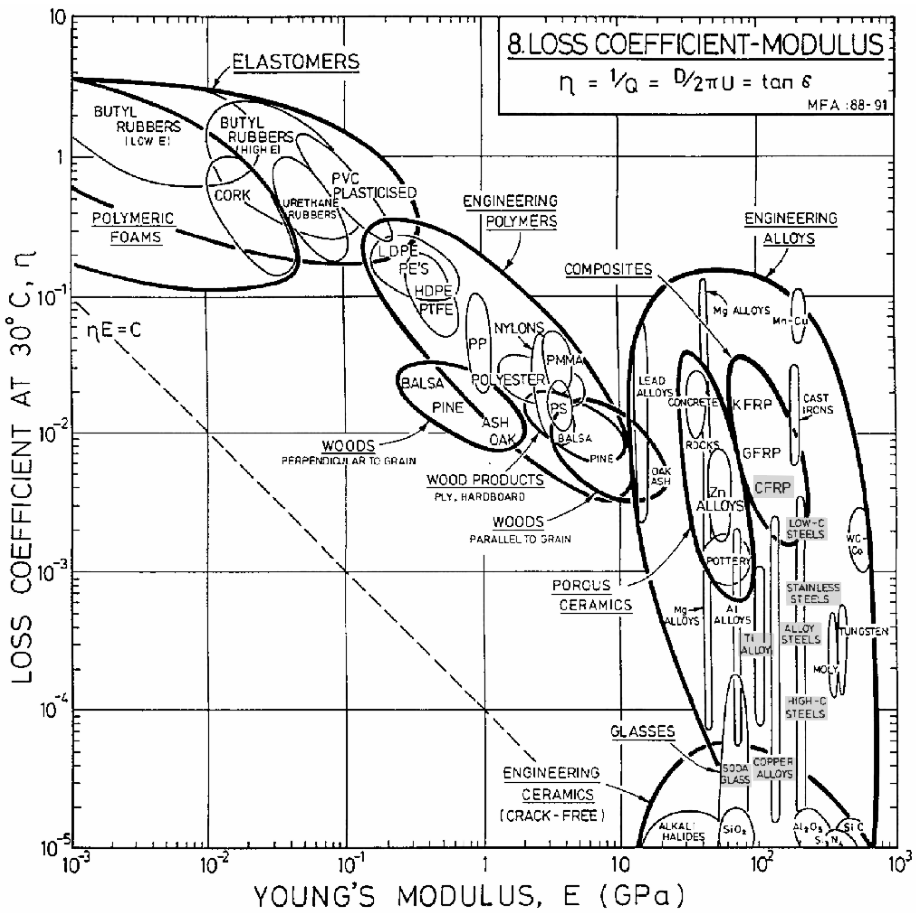

Figure 3. Loss coefficient η (from Ashby [5])
(NB. highlighted materials are of particular interest)

3.3 Surface finish

Ideally the surface finish for the blade is to be highly reflective and steel-like in colour *("... a large shiny square-sided kerosene ... a great flash of quivering sunlight ..." [from McCarthy & Leonard [6]).* Agreed inequality constraints in this case, were that the blade should have a surface *colour* comparable with or more steel-like than dull grey aluminium and that the blade surface roughness, R_a, be finer than $0.8\mu m$.

The larger size *Blade* is to be permanently exhibited at an outside location within 1km of the sea. Corrosion resistance, and in particular the resistance to attack by a salt water and U-V light are significant considerations for selecting candidate blade materials. Ashby [5] gives a comparative ranking of the resistance of materials, to attack by salt water and U-V radiation. Glass and titanium are expected to offer excellent resistance to attack by salt water followed by the Carbon Fibre Reinforced Polymer (CFRP) that has excellent to good resistance and followed by aluminium alloy, alloy steels, and carbon steel which are expected to be good, poor, and bad respectively. All materials with the exception of CFRP have excellent resistance to U-V light.

3.4 Economic availability

Early investigations found that the original blade material (cold rolled high carbon steel) was not available in the required thickness due to limitations of the manufacturing process. A high strength low alloy steel, while not generally available at the required length, was likely to be available at a suitable size if requested before a mill run.

High strength titanium alloys such as those listed in Duncan & Hanson [7] were found to be difficult to obtain in the dimensions required for a blade. The most suitable grade of titanium, likely to be available at the required size, was found to be Ti6Al/4V. Ti6Al/4V is produced in small batch runs, obtaining a suitable piece for *Blade* is likely to require a worldwide search of potential suppliers and mills. Finding suitable materials for the flexible components in Len Lye's kinetic sculptures have in the past been a limiting factor governing the maximum size (Raine et al. [8]).

Inequality constraints on economic availability were defined for a nominally double-original size sculpture. These upper limits require a blade material cost of less than *NZ$500 per performance* and an availability within a 12 month lead time.

3.5 Obtaining the optimal design solution

The most promising candidate blade materials were selected for further consideration based on their ability to meet the design constraints derived in Sections 3.1 – 3.4. While the choice of material may be made purely in terms of structural properties, the decision as to whether the sculpture is viable at a certain scale is also an economic one. In searching for an optimum solution it is useful to formulate a *'primary design equation'* (following Johnson [4]) to maximise or minimise. In this case the *primary design equation parameter U,* to be maximised, is the number for *Blade* performances for a given expenditure i.e.

$$U = \frac{\text{Number of performances to failure}}{\text{material cost}} \quad (NZ\$^{-1}) \tag{4}$$

The relationship between the number of cycles to failure N_f, the mean failure strength σ_f, and the endurance limit σ_e, may be represented using an S-N diagram. Typical S-N curves for ferrous and titanium alloys exhibit a steep drop in the high stress cycle fatigue life range *(typically from 10^3 – 10^6 cycles)* and levelling off to approach a stress asymptote *(the fatigue limit)* at longer lives (Collins [9]). For high cycle fatigue *(beyond 10^3 cycles of stress)* of wrought steels it is common practice to construct a line on an log S - log N chart joining $0.8\sigma_u$ at 10^3 cycles and σ_e at 10^6 cycles to define the mean fatigue strength corresponding to any life between 10^3 and 10^6 cycles. Based on this analogy, the line from 10^3 – 10^6 cycles in the log S - log N chart is defined by the relationship

$$log(\sigma_f) = -\frac{1}{3} log \frac{0.8k_s\sigma_u}{\sigma_e} log(N_f) + log\frac{(0.8k_s\sigma_u)^2}{\sigma_e} \qquad 10^3 < N_f < 10^6 \tag{5}$$

Since the blade bending stress and natural frequency are a function of blade thickness, d, these may be substituted into Equation (4). Noting that for geometric similarity, the blade curvature, *R(x)* is proportional to blade length and, for the case where the ratio of the ultimate strength to endurance limit is assumed to be constant, it can be shown (Gooch [4]) that the number of performances to failure per dollar cost can be expressed as

$$U = k_1 \left[\frac{\gamma^{k_2}}{\mu^{\frac{1}{2}} g^{k_2} T} \right] \left[\frac{1}{l^{(k_2+2)}} \right] \left[\frac{1}{E^{\left(k_2 - \frac{3}{2}\right)} \rho^{\left(k_2 + \frac{1}{2}\right)} C_m} \right] \tag{6}$$

where: T is the time spent vibrating at the maximum stress amplitude
μ is a dimensionless frequency parameter (Naguleswaran [1])
γ is a dimensionless stability parameter (Naguleswaran [1])
C_m the material cost per kg
E is the elastic modulus
ρ is the material density

g is the gravitational acceleration

k_1 & k_2 are positive constants

The first term in Equation (5) concerns the stability and frequency parameters and the second term specifies the blade shape parameter, these terms are fixed as they pertain to the artists aesthetic requirements. The third term, the blade material properties index, concerns the blade material properties only and is to be maximised for the maximum number of performances to failure per dollar cost. From Equation (5), it may be noted that the number of performances per dollar expended on the blade material varies inversely as $l^{(k_2+2)}$. There will be a rapid increase in cost per performance and reduction in life with increasing size. While cost rapidly becomes prohibitive, the limit on cost per performance *(NZ$500)* was not found (Gooch [3]) to be the limiting factor because the optimum solution is also constrained by availability of materials.

At the theoretical limiting scale the maximum bending stresses in the blade are approaching the yield stress, and the blade will be expected to complete not more than one performance before failure despite satisfying the buckling stability criterion. For the case of the titanium alloy and low alloy steel materials, the limit on blade length was calculated to be 6.0 and 4.7m respectively.

Figure 4. Public exhibition of Blade at Pukekura Park *(Bryan James, Govett-Brewster Art Gallery, New Zealand)*

The selected material for the blade is titanium alloy 6Al4V. The surface of the titanium blade was finished using a belt sanding process. The resulting surface roughness was *0.8μm* and the machining grain ran along the length dimension of the blade. The final blade thickness after the surface finishing process was *5.533mm*. Using the equality constraint for stability, Equation (2), results in a blade length *3.355m*.

Hemple & Hillnhagen [10] give empirical S-N data for flat Ti6Al/4V specimens in the annealed condition. The maximum calculated reversed bending stress at the fixed end of the *3.355m* Ti6Al/4V blade was *383MPa* and the corresponding number of cycles to failure, using Hemple & Hillnhagen [10], was predicted to be *2.05* x 10^5. *2.05* x 10^5 cycles corresponds to ***261*** performances before failure and a blade material cost of ***NZ$36*** per performance.

4. Discussion

Carbon Fibre Reinforced Polymer (CFRP) was found to be the most promising material in terms of meeting the strength of materials requirements for a blade. While this analysis predicted that a double size CFRP blade would be expected to have an infinite working life, this material was found to be unsuitable due to poor reverberant qualities.

From a sound quality perspective, the best material group investigated was glass. While glass was eliminated due to poor structural performance *(a low material properties index)* further consideration of this material is warranted when building similar sculptures. The final specification titanium alloy was found to have slightly better reverberant qualities than alloy steel, hence better meets the acoustic qualities for the sculpture.

The fatigue life for the scaled sculpture has been investigated using two independent methods. A theoretical model based on simple S-N relationships, Equation (5), yielded a similar result to the prediction using experimental results from Hemple & Hillnhagen [10].

The specified titanium alloy blade material has excellent corrosion resistance due to the fact that it is intrinsically very reactive (Duncan & Hanson [7]). Whenever fresh titanium alloy is exposed to an environment containing oxygen, it immediately acquires a thin tenacious oxide film.

The larger size version of Len Lye's Blade was built and tested at the University of Canterbury. Figure 4 shows the work at a public exhibition in Pukekura Park, New Plymouth, New Zealand.

5. Conclusion

The best possible solution satisfying the design requirement specification *(including specifications from Len Lye)* for the kinetic sculpture *Blade* has been obtained by the selection of an optimal blade material.

The two best candidate material groups identified for the blade material were found to be high strength titanium alloy and low alloy steel. A variation study of the number of kinetic performances per dollar expended gave a quantitive measure of the suitability of these materials and showed that titanium alloy is almost six times cheaper for a nominally double size work when considered on a cost per performance basis.

The optimal design of the kinetic sculpture Blade, within the artists brief, requires the sculpture to have a titanium alloy (6Al/4V) blade with a nominal blade length of 3.355m. The larger *Blade* is expected to fail due to reversed bending fatigue at the fixed end after *260* performances.

The selected titanium alloy has good reverberant properties and is expected to result in the acceptable sound qualities for the sculpture. This material also has particularly good resistance to attack by the salt air and will have a self-repairing quality if scratched in place.

Availability of a suitable blade material was found to be the constraint limiting the realisable size of *Blade*.

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Shayne Gooch, PhD, Lecturer
Department of Mechanical Engineering
University of Canterbury, Private Bag 4800,
Christchurch 8020, New Zealand
Telephone: +64 33642987 x 7160, Telefax+64 33642078
E-mail: shayne.gooch@canterbury.ac.nz